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Q-FUZZY DERIVATIONS ON N-PICTURE FUZZY SOFT SUBGROUP STRUCTURES

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ABSTRACT

In this paper, the notion of Q-Fuzzy derivations on N-Picture fuzzy soft subgroups is introduced, and related properties are investigated. Characterizations of Q-Fuzzy derivations on N-Picture fuzzy soft subgroup are established, and how images or inverse images of N-Picture Fuzzy soft subgroups become N-Picture fuzzy soft subgroups studied.

KEYWORDS: Soft Set, Q-fuzzy Set, N-Picture Fuzzy Soft Set, N-Picture Fuzzy Soft Group, Image, Inverse Image, Supremum & Infimum

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1. INTRODUCTION

A fuzzy set was first introduced by Zadeh [18] and then the fuzzy sets have been used in there consideration of classical mathematics. Yuan.et.al.[17] introduced the concept of fuzzy subgroup with thresholds. A fuzzy subgroup with thresholds l and μ is also called a (λ, μ)-fuzzy subgroup. A.Solairaju and R.Nagarajan introduced the concept of Structures of Q- fuzzy groups [15]. A.Solairaju and R. Nagarajan studied some structural properties of upper Q-fuzzy index order, with upper Q- fuzzy subgroups[16]. Such inaccuracies are associated with the membership function that belongs to [0,1]. Through membership function, we obtain information which makes possible for us to reach the conclusion. The fuzzy set theory becomes a strong area of making observations in different areas like medical science, social sciences, engineering, management sciences, artificial intelligence, robotics, computer networks, decision making and so on. Due to unassociated sorts of unpredictability occurring in different areas of life like economics, engineering, medical sciences, management sciences, psychology, sociology, decision making and fuzzy set as noted and often effective mathematical instruments have been offered to make, be moving in and grip those unpredictability. Since the establishment of fuzzy set, several extensions have been made such as Atanassov's([3], [4], [5]. [6])work on intuitionistic fuzzy set (IFSs) was quite remarkable as he extended the concept of FSs by assigning non-membership degree say "N(x)" along with membership degree say "P(x)" with condition that $0 \leq P(x)+N(x) \leq 1$. Since last few decades, the IFS has been explored by many researchers and successfully applied to many practical fields like medical diagnosis, clustering analysis, decision making pattern recognition [3, 4, 5, 6]. Strengthening the concept IFS suggest Pythagorean fuzzy sets which somehow enlarge the space of positive membership and negative membership by introducing some new condition that $0 \leq P^2(x) + N^2(x) \leq 1$. Molodtsov [12] introduced the concept of soft sets that can be seen as a new

mathematical theory for dealing with uncertainty. The soft set theory has been applied to many different fields with great success. Maji.et.al. ([7],[8],[9]) worked on theoretical study of soft sets in detail, and presented an application of soft set in the decision making problem using the reduction of rough sets. In this paper, the notion of Q-Fuzzy derivations on N-Picture Fuzzy Soft Subgroups is introduced, and related properties are investigated. Characterizations of Q-Fuzzy derivations on N-Picture Fuzzy Soft Subgroup are established, and how images or inverse images of N-Picture fuzzy soft subgroups become N-Picture fuzzy soft subgroups studied.

2. PRELIMINARIES

Let I be a Closed Unit Interval, (i.e.) I = [–1, 0]. By an interval number we mean a closed sub interval $\tilde{a} = [\bar{a}, \overset{+}{a}]$ of I, where $0 \le \bar{a} \le \overset{+}{a} \le 1$. Denote by D[–1, 0] the set of all interval numbers. Let us define what is known as refined minimum (briefly, r min) of two elements in D[0,1]. We also define the symbols "≤", "≥", "=" in case of two elements in D[0,1]. Consider two interval numbers $\tilde{a}_1 = [\bar{a}_1, \overset{+}{a}_1]$ and $\tilde{a}_2 = [\bar{a}_2, \overset{+}{a}_2]$.

Then, $r\min\{\tilde{a}_1, \tilde{a}_2\} = [\min\{a_1^-, a_2^-\}, \min\{a_1^+, a_2^+\}]$, $\tilde{a}_1 \ge \tilde{a}_2$ if and only if $a_1^- \ge a_2^-$ and $a_1^+ \ge a_2^+$, and similarly we may have $\tilde{a}_1 \le \tilde{a}_2$ and $\tilde{a}_1 = \tilde{a}_2$. To say $\tilde{a}_1 > \tilde{a}_2$ (respectively $\tilde{a}_1 < \tilde{a}_2$) we mean $\tilde{a}_1 \ge \tilde{a}_2$ and $\tilde{a}_1 \ne \tilde{a}_2$ (respectively $\tilde{a}_1 \le \tilde{a}_2$ *and* $\tilde{a}_1 \ne \tilde{a}_2$). Let $\tilde{a}_i \in D[0, 1]$ we have $i \in \wedge$.

We define $r \inf_{i \in \wedge} \tilde{a}_i = [\inf_{i \in \wedge} a_i^-, \inf_{i \in \wedge} a_i^+]$ *and* $r \sup_{i \in \wedge} \tilde{a}_i = [\sup_{i \in \wedge} a_i^-, \sup_{i \in \wedge} a_i^+]$.

An interval-valued N-Fuzzy set (briefly IVNF set) $\tilde{M}_A$ defined on a non-empty set X is given by $\tilde{M}_A = \{(x, [M^-_A(x), M^+_A(x)] / x \in X\}$, which is briefly denoted by $\tilde{M}_A = [M^-_A, M^+_A]$ where M^-_A *and* M^+_A are two N-Fuzzy sets in X such that $M^-_A(x) \le M^+_A(x)$ for all $x \in X$. For any IVNF set $\tilde{M}_A$ on X and $x \in X$, $\tilde{M}_A(x) = [M^-_A(x), M^+_A(x)]$ is called the degree of membership of an element x to $\tilde{M}_A$, in which $M^-_A(x)$ *and* $M^+_A(x)$ are refixed to as the lower and upper degrees, respectively of membership of x to $\tilde{M}_A$.

Definition 2.1: A fuzzy set μ in a universe X is a mapping μ : X→ [0,1].

Definition 2.2: Let U be any Universal set, E set of parameters and A⊆ E. Then a pair (K,A) is called soft set over U, where K is a mapping from A to 2^U, the power set of U.

Example 2.3: Let X={c_1,c_2,c_3} be the set of three cars and E={costly(e_1), metallic colour(e_2), cheap(e_3)}be the set of parameters, where A={e_1,e_2}⊂E. Then (K,A)={K(e_1)={c_1,c_2,c_3},K(e_2)={c_1,c_2}} is the crisp soft set over X.

Definition 2.4:Let Q and G be a set and a group respectively. A mapping A:G×Q→ [0,1] is called Q-fuzzy set in G. For any Q-fuzzy set A in G and t ∈ [0,1]. The set U (A:t) ={x∈G/ A(x,q)≥t, q∈Q} which is called an upper cut of A.

Definition 2.5: Let U be an initial universe. Let P (U) be the power set of U, E be the set of all parameters and A⊆ E. A soft set (f_A, E) on the universe U is defined by the set of order pairs (f_A, E) = {(e, f_A (e)): e∈ E, f_A∈ P (U)} where

f_A : E → P (U) such that f_A (e) = ϕif e ∉A. Here f_A is called an approximate function of the soft set.

Example 2.6: Let U = {u_1,u_2,u_3,u_4} be a set of four shirts and E = {white(e_1),red(e_2),blue(e_3)} be a set of parameters. If A = {e_1, e_2} ⊆ E. Let $f_A(e_1)$ = {u_1, u_2, u_3, u_4} and $f_A(e_2)$= {u_1,u_2,u_3}. Then we write the soft set (f_A, E)= {(e_1, {u_1, u_2, u_3, u_4}), (e_2,{ u_1,u_2,u_3})} over U which describe the "colour of the shirts" which Mr. X is going to buy. We may represent the soft set in the following form: U = {(e_1,u_1), (e_2,u_1), (e_1, u_2), (e_2,u_2), (e_1,u_3), (e_2, u_3), (e_1, u_4)}.

Definition 2.7: Let U be the universal set, E set of parameters and A⊂ E. Let K(X) denote the set of all fuzzy subsets of U. Then a pair (K,A) is called fuzzy soft set over U, where K is a mapping from A to K(U).

Example 2.8:Let U={c_1,c_2,c_3} be the set of three cars and E={costly(e_1),metallic color(e_2), cheap(e_3)} be the set of parameters, where A={e_1,e_2} ⊂ E. Then (K,A)={K(e_1)={c_1/0.6,c_2/0.4,c_3/0.3}, K(e_2)={ c_1/0.5,c_2/0.7,c_3/0.8}} is the fuzzy soft set over U denoted by F_A.

Definition 2.9: Let K_A be a fuzzy soft set over U and α be a subset of U then upperα - inclusion of K_A denoted by $K^{\alpha}{}_A$ = { x∈ A /K(x) ≥ α }. Similarly $K^{\alpha}{}_A$ = {x∈ A / K(x) ≤ α} is called lower α-inclusion of K_A.

Definition 2.10: Let K_A and G_B be fuzzy soft sets over the common universe U and ψ : A → B be a function. Then fuzzy soft image of K_A under ψ over U denoted by ψ(K_A) is a set-valued function, where ψ(K_A):B→ 2^U defined by ψ(K_A) (b)={∪{K(a) / a∈ A and ψ (a)=b}, if ψ^{-1}(b)≠ φ} for all b∈ B, the soft pre-image of G_B under ψ over U denoted by $\psi^{-1(}$$G_B$) is a set-valued function, where$\psi^{-1(}$$G_B$) : A → 2^U defined by ψ^{-1}(G_B)(b) = G(ψ(a)) for all a∈ A. Then fuzzy soft anti-image of K_A under ψ over U denoted by ψ(K_A) is a set-valued function, where ψ(K_A):B → 2^U defined byψ^{-1}(K_A)(b)={∩{K(a) / a∈ A and ψ (a)=b}, if ψ^{-1}(b) ≠ φ for all b∈ B.

Definition 2.11:A Picture fuzzy set (PFS) A on a universe X is an object of the form A={(x,μ_A(x),η_A(x),ν_A(x))|x ∈ X}where μ_A(x) ∈[0,1] is called the degree of positive membership(PM) of x in A, η_A(x) ∈[0,1] is called the degree of neutral membership (NeuM) of x in A, ν_A(x) ∈ [0,1] is called the degree of negative membership (NM) of x in A. μA (x),η_A(x),ν_A (x) must satisfy the condition μ_A (x)+η_A(x)+ν_A(x) ≤1∀x ∈ X. Then ∀x ∈ X, 1−(μ_A (x)+η_A (x)+ν_A(x)) is called the degree of refusal membership of "x" in A.

Definition 2.12: A negative picture fuzzy (N- picture fuzzy) soft set $\boldsymbol{\mathcal{A}}$ on the universe of discourse$X$ is defined as $\boldsymbol{\mathcal{A}} = \langle x, \bar{\delta}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(x)\rangle, x \in \mathcal{X}$, where δ, I, Δ : $\boldsymbol{\mathcal{X}}$→ [−1, 0] and − 1 ≤ $\bar{\delta}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(x)$≤ 0.

Definition 2.13: A N- picture Fuzzy softset $\boldsymbol{\mathcal{A}}$ over the universe $\boldsymbol{\mathcal{X}}$ is said to be null or empty N-picture fuzzy soft set if $\delta_{\mathcal{A}}(x) = -1, I_{\mathcal{A}}(x) = -1, \Delta_{\mathcal{A}}(x) = 0$ for all $x \in \mathcal{X}$. It is denoted by -1_N.

Definition 2.14: A N- picture fuzzy softset $\boldsymbol{\mathcal{A}}$over the universe $\boldsymbol{\mathcal{X}}$ is said to be absolute (universe) N-picture fuzzy soft set if

$\delta_{\mathcal{A}}(x) = 0, I_{\mathcal{A}}(x) = 0, \Delta_{\mathcal{A}}(x) = -1$ for all $x \in \mathcal{X}$. It is denoted by 0_N.

3. Q-FUZZY DERIVATION OF N-PICTURE FUZZY SOFT GROUP

In what follows let X denote a group unless otherwise specified.

Definition 3.1: Let X be a non-empty set. A Q-Fuzzy derivation of an N-Picture Fuzzy Soft set A in a set X is a structure $A = \{((x,q), \tilde{M}_A(x,q), N(x,q)) / x \in X,\ q \in Q\}$which briefly denoted by $A = (\tilde{M}_A, N_A)$ where

$\tilde{M}_A = [M^-{}_A,\ M^+{}_A]$ is on IVNF soft set in X and N_A is an N-Fuzzy soft set in X.

Denoted by $D^N(X)$ the families of co-fuzzy derivations of N-Picture fuzzy soft set X.

Definition 3.2: A Q-Fuzzy derivation of N-Picture Fuzzy Soft set $A = (\tilde{M}_A, N_A)$ in X is called a N-Picture fuzzy soft group of X if it satisfies: for all $x, y \in X$ *and* $q \in Q$.

$$(QNPFSG-1): \tilde{M}_A(xy,q) \geq r\ \min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\}$$

$$(QNPFSG-2): \tilde{M}_A(x^{-1},q) \geq \tilde{M}_A(x,q)$$

$$(QNPFSG-3): N_A(xy,q) \leq \max\{N_A(x,q),\ N_A(y,q)\}$$

$$(QNPFSG-4): N_A(x^{-1},q) \leq N_A(x,q).$$

Example 3.3: Let X be the Klein's four group. We have $X = \{e, a, b, ab\}$ where $a^2 = e = b^2$ *and* $ab = ba$.

We define $\tilde{M}_A = [M^-{}_A,\ M^+{}_A]$ and N_A by

$$\tilde{M}_A = \begin{pmatrix} e & a & b & ab \\ [-0.7,-0.4] & [-0.8,-0.5] & [-0.1,-0.3] & [-0.1,-0.7] \end{pmatrix} \text{ and}$$

$$N_A = \begin{pmatrix} e & a & b & ab \\ -0.2 & -0.4 & -0.6 & -0.7 \end{pmatrix}.$$

Then $A = (\tilde{M}_A, N_A)$ is Q-Fuzzy derivations of N-Picture Fuzzy Soft group.

Example 3.4: Let X be a non-trivial group and define an IVNF set $\tilde{M}_B = [M^-{}_B,\ M^+{}_B]$ and an N-Fuzzy set K by

$$\tilde{M}_B(e,q) = [S_c,\ t_c] \text{ and } \tilde{M}_B(x,q) = [S,\ t] \text{ for all } x \neq e \text{ where } [S_c,\ t_c] < [S,\ t] \text{ in } D[-1,0],$$

$K(e) = t_c$ *and* $K(x) = t$ *for all* $x \neq e$ *where* $r_c > r$ *in* $[-1,0]$ and 'e' is the identity element of X, then $B = (\tilde{M}_B, k)$ is Q-Fuzzy derivations of N-Picture Fuzzy Soft group of X.

Propositions 3.5: Let $A = (\tilde{M}_A, N_A)$ be Q-Fuzzy derivations of N-Picture Fuzzy Soft group of X, then $\tilde{M}_A(x^{-1},q) = \tilde{M}_A(x,q)$ *and* $N_A(x^{-1},q) = N_A(x,q)$ *for all* $x \in X,\ q \in Q$.

Proof: For any $x \in X$, we have $\tilde{M}_A(x,q) = \tilde{M}_A((x^{-1})^{-1},q) \geq \tilde{M}_A(x^{-1},q) \geq \tilde{M}_A(x,q)$ *and* $N_A(x,q) = N_A((x^{-1})^{-1},q) \leq N_A(x^{-1},q) \leq N_A(x,q).$

Propositions 3.6: Let $A = (\tilde{M}_A, N_A)$ be Q-Fuzzy derivations of N-Picture Fuzzy Soft group of X, then $\tilde{M}_A(e,\ q) \geq \tilde{M}_A(x,q)\ and\ N_A(e,\ q) \leq N_A(x,q)\ \ for\ all\ x \in X,\ q \in Q,$ where 'e' is the identity element of X.

Proof: For any $x \in X$, using Proposition-1,

We have, $\tilde{M}_A(e,q) = \tilde{M}_A(xx^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(x^{-1},q)\} = \tilde{M}_A(x,q)\ and$

$N_A(e,q) = N_A(xx^{-1},q) \leq \max\{N_A(x,q),\ N_A(x^{-1},q)\} = N_A(x,q).$

This completes the proof.

Propositions 3.7: Let $A = (\tilde{M}_A, N_A)$ be Q-Fuzzy derivations of N-Picture Fuzzy Soft group of X. For any $x, y \in X$, if $\tilde{M}_A(xy^{-1},q) = \tilde{M}_A(e)$ and $N_A(xy^{-1},q) \leq N_A(e,q)$, then $\tilde{M}_A(x,q) = \tilde{M}_A(y,q)$ and $N_A(x,q) = N_A(y,q)$.

Proof: Let $x \in X\ and\ q \in Q$ be such that $\tilde{M}_A(xy^{-1},\ q) = \tilde{M}_A(e,q)\ and\ N_A(xy^{-1},\ q) \leq N_A(e,q).$

Using Proposition – 2,

We get, $\tilde{M}_A(x,q) = \tilde{M}_A((xy^{-1})y,q) \geq r\min\{\tilde{M}_A(e,q),\ \tilde{M}_A(y,q)\} = \tilde{M}_A(y,q)$ and

$N_A(x,q) = N_A((xy^{-1})y,q) \leq \max\{N_A(e,q),\ N_A(y,q)\} = N_A(y,q),$ for all $x \in X\ and\ q \in Q.$

$Similarly,\ \ \tilde{M}_A(y,q) \geq \tilde{M}_A(x,q)\ and\ N_A(y,q) \leq N_A(x,q).$

Hence the proof.

Question 3.8: For any $x \in X,\ if\ \tilde{M}(y,q) > \tilde{M}(x,q)\ and\ N(y,q) < N(x,q),$ then the inequalities are $\tilde{M}_A(xy,q) = \tilde{M}_A(x,q) = \tilde{M}_A(yx,q)\ and\ N_A(xy,q) = N_A(x,q) = N_A(yx,q)\ true$?

The following example provides a negative answer to the question – 3.8.

Example 3.9: In the Klein's four group $X = \{e,\ a,\ b,\ ab\}$ we define $\tilde{M}_A = [M^-{}_A,\ M^+{}_A]$ and N_A by

$$\tilde{M}_A = \begin{pmatrix} e & a & b & ab \\ [-0.4,-0.9] & [-0.2,-0.7] & [-0.3,-0.8] & [-0.1,-0.9] \end{pmatrix} and$$

$$N_A = \begin{pmatrix} e & a & b & ab \\ -0.7 & -0.4 & -0.6 & -0.7 \end{pmatrix}.$$

Then $A = (\tilde{M}_A, N_A)$ is a Q-Fuzzy derivation of N-Picture fuzzy soft group of X. Note that,

$$\tilde{M}_A(b,q) = [-0.3,-0.8] > [-0.4, 0.9] = \tilde{M}_A(a,q)\ and\ N_A(b,q) = -0.6 < -0.4 = N_A(a,q).$$

But, $\tilde{M}_A(ab,q) = [-0.1, -0.9] \neq [-0.4, -0.8]$.

4. PROPERTIES OF Q-FUZZY DERIVATION OF N-PICTURE FUZZY SOFT GROUP

In this section, we provide characterizations of N-Picture fuzzy soft groups.

Theorem4.1: An N-Picture fuzzy soft set $A = (\tilde{M}_A, N_A)$ in X is Q-Fuzzy derivation of N-Picture fuzzy soft group [QNPFSG] of X if and only if satisfies

(i) $\tilde{M}_A(xy^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\, \tilde{M}_A(y,q)\}$

(ii) $N_A(xy^{-1},q) \leq \max\{N_A(x,q),\, N_A(y,q)\}$ for all $x \in X$ *and* $q \in Q$.

Proof: Assume $A = (\tilde{M}_A, N_A)$ is an [QNPFSG] of X and let $x \in X$. Then,

$$\tilde{M}_A(xy^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\, \tilde{M}_A(y,q)\}$$

$$= r\min\{\tilde{M}_A(x,q),\, \tilde{M}_A(y,q)\} \text{ and}$$

$$N_A(xy^{-1},q) \leq \max\{N_A(x,q),\, N_A(y^{-1},q)\}$$

$$= \max\{N_A(x,q),\, N_A(y,q)\}$$

By Proposition – 3.5,conversely, suppose that (i) and (ii) are valid.

If we take y = x in (i) and (ii), then

$$\tilde{M}_A(e,q) = \tilde{M}_A(xx^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\, \tilde{M}_A(y,q)\} = \tilde{M}_A(x,q) \text{ and}$$

$$N_A(e,q) = N_A(xx^{-1},q) \leq \max\{N_A(x,q),\, N_A(x,q)\} = N_A(x,q).$$

It follows from (i) and (ii) that

$$\tilde{M}_A(y^{-1},q) = \tilde{M}_A(ey^{-1},q) \geq r\min\{\tilde{M}_A(e,q),\, \tilde{M}_A(y,q)\} = \tilde{M}_A(y,q) \text{ and}$$

$$N_A(y^{-1},q) = N_A(ey^{-1},q) \leq \max\{N_A(e,q),\, N_A(y,q)\} = N_A(y,q).$$

So that,

$$\tilde{M}_A(xy,q) = \tilde{M}_A(x(y^{-1})^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\, \tilde{M}_A(y^{-1},q)\}$$

$$\geq r\min\{\tilde{M}_A(x,q),\, \tilde{M}_A(y,q)\} \quad \text{and}$$

$$N_A(xy,q) = N_A(x(y^{-1})^{-1},q) \leq \max\{N_A(x,q),\, N_A(y^{-1},q)\}$$

$$\leq \max\{N_A(x,q),\, N_A(y,q)\}$$

Therefore, $A=(\tilde{M}_A, N_A)$ is an [QNPFSG] of X.

Theorem 4.2: If $A=(\tilde{M}_A, N_A)$ is a Q-Fuzzy derivation of N-Picture fuzzy soft group [QNPFSG] of X then the set $S = \{ x\in X / \tilde{M}_A(x,q) = \tilde{M}_A(e,q),\ N_A(x,q) = N(e,q) \}$ is a subgroup of X.

Proof: Let $x, y \in S$, *then* $\tilde{M}(x,q) = \tilde{M}(e,q) = \tilde{M}_A(y,q)$ *and* $N_A(x,q) = N_A(e,q) = N_A(y,q)$.

It follows from theorem – 1 that

$$\tilde{M}_A(xy^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\} = \tilde{M}_A(e,q)\ and$$

$$N_A(xy^{-1},q) \leq \max\{N_A(x,q),\ N_A(y,q)\} = N_A(e,q).$$

So from Proposition – 3.6 that

$$\tilde{M}_A(xy^{-1},q) = \tilde{M}_A(e,q)\quad and\quad N_A(xy^{-1},q) = N_A(e,q).$$

$$Hence,\quad xy^{-1} \in S\ and\ q \in Q.$$

So, S is a subgroup of X.

Definition 4.3: Let $A=(\tilde{M}_A, N_A)$ be an N-Fuzzy soft set in a set X, $r \in [0,1]\ and\ [s,t] \in D[-1,0]$. The set $\bigcup(A:[s,t],r) = \{x \in X / \tilde{M}_A(x,q) \geq [s,t],\ N(x,q) \leq r\}$ is called the Q-Fuzzy derivation of N-Picture fuzzy level soft set of A.

Theorem 4.4: For an N-fuzzy soft set $A=(\tilde{M}_A, N_A)$ in X, the following are equivalent.

(i) $A=(\tilde{M}_A, N_A)$ is a Q-Fuzzy derivation of N-Picture fuzzy soft group [QNPFSG] of X.

(ii) The non-empty Q-Fuzzy derivation of N-Picture fuzzy level soft set of A is a subgroup of X.

Proof: Assume that A is a QNPFSG of X. Let $x, y \in \bigcup(A:[s,t],r)$, for all $r \in [0,1]\ and\ [s,t] \in D[-1,0]$.

Then,

$$\tilde{M}_A(x,q) \geq [s,t],\quad N_A(x,q) \leq r,$$

$$\tilde{M}_A(y,q) \geq [s,t],\quad N_A(y,q) \leq r.$$

It follows from theorem-1 that

$$\tilde{M}_A(xy^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\} \geq [s,t]\ and$$

$$N_A(xy^{-1},q) \leq \max\{N_A(x,q),\ N_A(y,q)\} = r.$$

So that, $x, y^{-1} \in \bigcup(A:[s,t],r)$.

Therefore, the non-empty N-Picture fuzzy level soft set of A is a subgroup of X.

Conversely, let $r \in [-1,0]\, and\ [s,t] \in D[-1,0]$ be such that $\bigcup(A:[s,t],r) \neq \phi\ \ and\ \bigcup(A:[s,t],r)$

is a subgroup of X.

Suppose that Theorem 4.1 (i) is not true and Theorem 4.1 (ii) is valid, then there exists $[s_0,t_0] \in D[-1,0]$ and $a,b \in X$ such that

$$\tilde{M}_A(ab^{-1},q) \leq [s_0,t_0] \leq r\min\{\tilde{M}_A(a,q),\ \tilde{M}_A(b,q)\}\ \ and$$

$$N_A(ab^{-1},q) \leq \max\{N_A(a,q),\ N_A(b,q)\}.$$

It follows that $a,b \in \bigcup(A:[s_0,t_0],r),\ \max\{N_A(a,q),\ N_A(b,q)\}$

But $a,b^{-1} \notin \bigcup(A:[s_0,t_0],\ \max\{N_A(a,q),\ N_A(b,q)\})$.

This is a contradiction.

If Theorem –4.1 (i) is true and Theorem –4.1 (ii) is not valid, then

$$\tilde{M}_A(ab^{-1},q) \geq r\min\{\tilde{M}_A(a,q),\ \tilde{M}_A(b,q)\}\ \ and$$

$$N_A(ab^{-1},q) > r_0 \geq \max\{N_A(a,q),\ N_A(b,q)\}.$$

$$for\, some\ r_0 \in [-1,0]\ and\ a,b \in X.$$

Thus $a,b \in \bigcup(A:\ r\min\{\tilde{M}_A(a,q),\ \tilde{M}_A(b,q),r_0\}$

But, $a,b^{-1} \notin \bigcup(A:\ r\ \min\{\tilde{M}_A(a,q),\ \tilde{M}_A(b,q),r_0\})$.

This is a contradiction.

Assume that there exist $[s_0,t_0] \in D[-1,0]$, $r_0 \in [-1,0]\ and\ a,b \in X\ \ such\ that$

$$\tilde{M}_A(ab^{-1},q) \leq [s_0,t_0] \leq r\min\{\tilde{M}_A(a,q),\ \tilde{M}_A(b,q)\}\ \ and$$

$$N_A(ab^{-1},q) > r_0 \geq \max\{N_A(a),\ N_A(b)\}.$$

Then $a,b \in \bigcup(A:[s_0,t_0],r_0)\ but\ \ ab^{-1} \notin \bigcup(A:[s_0,t_0],r_0)$

This is contradiction. Hence (i) and (ii) of Theorem –4.1 are valid.

Therefore 'A' is QNPFSG of X.

Definition 4.5: Let X and Y be given classical set. A mapping $\chi : X \to Y$ induces two mappings $R_\chi : R(X) \to R(Y),\ \ A \mapsto R_\chi(A)$ and $R_\chi^{-1} : R(Y) \to R(X),\ \ B \mapsto R_{\chi^{-1}}(B)$, where $R_\chi(A)$ is given by

$$R_\chi(\tilde{M}_A)(y,q) = \begin{cases} r\sup\limits_{y=\chi(x)} \tilde{M}_A(x,q) & if\ \ \chi^{-1}(y,q) \neq \phi \\ [0,0] & otherwise \end{cases}$$

$$R_\chi(N_A)(y,q) = \begin{cases} \inf\limits_{y=\chi(x)} N_A(x,q) & if\ \ \chi^{-1}(y,q) \neq \phi \\ -1 & otherwise \end{cases}$$

For all $y \in Y$ and $R_\chi^{-1}(B)$ is defined by $R_\chi^{-1}(\tilde{M}_B)(x,q) = \tilde{M}_B(\chi(x))$ and $R_\chi^{-1}(\rho(x)) = \rho(\chi(x))$ for all $x \in X$, then the mapping R_χ $(respectively\ R_\chi^{-1})$ is called N-Picture fuzzy soft transformation induced by χ.

Theorem 4.6: For a homomorphism $\chi : X \to Y$ of groups, let $R_\chi R(X) \to R(Y)$ and $R_\chi^{-1} : R(Y) \to R(X)$ be the N-Picture fuzzy soft transformation and inverse cubic transformation, respectively induced by χ.

(i) If $A = (\tilde{M}_A, N_A) \in R(X)$ is QNPFSG of X, which has the Picture fuzzy soft property, then $R_\chi(A)$ is QNPFSG of Y.

(ii) If $B = (\tilde{M}_B, \rho) \in R(Y)$ is QNPFSG of Y, which has the Picture fuzzy soft property, then $R_\chi^{-1}(B)$ is QNPFSG of X.

Proof: (i) Given $\chi(x), \chi(y) \in \chi(X),\ let\ x_0 \in \chi^{-1}(\chi(x))\ and\ y_0 \in \chi^{-1}(\chi(y))$ be such that

$$\tilde{M}_A(x_0,q) = r\sup_{a \in \chi^{-1}(\chi(x))} \tilde{M}_A(a,q), \qquad N_A(x_0,q) = \inf_{a \in \chi^{-1}(\chi(x))} N_A(a,q) \text{ and}$$

$$\tilde{M}_A(y_0,q) = r\sup_{b \in \chi^{-1}(\chi(y))} \tilde{M}_A(b,q), \qquad N_A(y_0,q) = \inf_{b \in \chi^{-1}(\chi(y))} N_A(b,q) \text{ respectively.}$$

Then,

$$R_\chi(\tilde{M}_A)\big(\chi(x)\chi(y), q\big) = r\sup_{(z,q) \in \chi^{-1}(\chi(x)\chi(y))} \tilde{M}_A(z,q) \geq \tilde{M}_A(x_0 y_0, q)$$

$$\geq r\ \min\left\{\tilde{M}_A(x_0,q),\ \tilde{M}_A(y_0,q)\right\}$$

$$= r\ \min\left\{ r\sup_{a \in \chi^{-1}(\chi(x))} \tilde{M}_A(a,q),\ r\sup_{b \in \chi^{-1}(\chi(y))} \tilde{M}_A(b,q) \right\}$$

$$= r\ \min\left\{R_\chi(\tilde{M}_A)(\chi(x)),\ R_\chi(\tilde{M}_A)(\chi(y))\right\}$$

$$R_\chi(\tilde{M}_A)\left(\chi^{-1}(x),q\right) = r\sup_{z\in\chi^{-1}\left(\chi^{-1}(x)\right)} \tilde{M}_A(z,q) \ge \tilde{M}_A(x_0^{-1},q) \ge \tilde{M}_A(x_0,q) = R_\chi(\tilde{M}_A)\left(\chi(x)\right).$$

$$R_\chi(N_A)\left(\chi(x)\chi(y),q\right) = \inf_{(z,q)\in\chi^{-1}(\chi(x)\chi(y))} N_A(z,q)$$

$$\le N_A(x_0 y_0,q)$$

$$\le \max\{N_A(x_0,q),\ N_A(y_0,q)\}$$

$$= \max\left\{\inf_{a\in\chi^{-1}(\chi(x))} N_A(a,q),\ \inf_{b\in\chi^{-1}(\chi(y))} N_A(b,q)\right\}$$

$$= \max\{R_\chi(\chi(x)),\ R_\chi(\chi(y))\}$$ and

$$R_\chi(N_A)\left(\chi^{-1}(x),q\right) = \inf_{(z,q)\in\chi^{-1}\left(\chi^{-1}(x)\right)} N_A(z,q)$$

$$\le N_A(x_0^{-1},q) \le N_A(x_0,q) = R_\chi(N_A)\left(\chi(x)\right).$$

Therefore $R_\chi(A)$ is a QNPFSG of Y.

(ii) For any $x, y \in X$, *we have*

$$R_\chi^{-1}(\tilde{M}_B)\left(xy,q\right) = \tilde{M}_B(\chi(xy,q))$$

$$= \tilde{M}_B(\chi(x)\chi(y),q))$$

$$\ge r\ \min\{\tilde{M}_B(\chi(x),q),\ \tilde{M}_B(\chi(y),q)\}$$

$$\ge r\ \min\{R_\chi^{-1}(\tilde{M}_B)(x,q),\ R_\chi^{-1}(\tilde{M}_B)(y,q)\}$$

$$R_\chi^{-1}(\tilde{M}_B)\left(x^{-1},q\right) = \tilde{M}_B(\chi(x^{-1},q)) \ge \tilde{M}_B(\chi(x,q)) = R_\chi^{-1}(\tilde{M}_B)(x,q)$$

$$R_\chi^{-1}(\rho)\left(xy,q\right) = \rho(\chi(xy),q) = \rho(\chi(x)\chi(y),q)$$

$$\le \max\{\rho(\chi(x),q),\ \rho(\chi(y),q)\}$$

$$\le \max\{R_\chi^{-1}(\rho(x,q),\ R_\chi^{-1}(\rho(y,q)\}$$ and

$$R_\chi^{-1}(\rho)\left(x^{-1},q\right) = \rho(\chi(x^{-1}),q) \le \rho(\chi(x),q) \le R_\chi^{-1}(\rho)(x,q).$$

Hence $R_\chi^{-1}(B)$ is a QNPFSG of X.

Note 4.7: Q-Fuzzy derivation of N-Picture fuzzy soft set A in X has the Picture fuzzy property if for any subset T of X there exists $x_0 \in T$ such that

$$\tilde{M}_A(x_0, q) = r \sup_{(x,q)\in T} \tilde{M}_A(x, q), \; N_A(x_0, q) = \inf_{(x_0,q)\in T} N_A(x_0, q).$$

5. CONCLUSIONS

Maji et al. [9] worked on the theoretical study of soft sets in detail and presented an application of soft set in the decision making problem using the reduction of rough sets. In this paper, the notion of Q-Fuzzy derivations on N-Picture fuzzy soft subgroups is introduced, and related properties are investigated. Characterizations of Q-Fuzzy derivations on N-Picture fuzzy soft subgroup are established, and how images or inverse images of N-Picture fuzzy soft subgroups become N-Picture Fuzzy Soft Subgroups studied

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